THE ORIENTATION OF MELT BANDS IN AN EXTENDING LITHOSPHERE

Hans Muhlhaus¹, Arash Mohajeri¹, Yaron Finzi¹ and Klaus Regenauer-Lieb²

¹ Earth Systems Science Computational Centre (ESSCC), School of Earth Sciences,
The University of Queensland, St. Lucia, QLD 4072
e-mail: h.muhlhaus@uq.edu.au

² School of Earth & Environment
The University of Western Australia, WA 5009
e-mail: klaus@cyllene.uwa.edu.au

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Abstract. Understanding how melt flow is organized on lithospheric scales is an important goal in modelling magma dynamics and tectonic processes such as rifting. Observations from nature and experimental data indicate that strain is highly localized in crustal and mantle shear zones and that melt processes may further enhance strain-localization of these structures. The coupling of lithospheric shear zones and melt flow is a key aspect in these processes and an outstanding challenge for both numerical modellers and geologists in recent years. We present an outline of the equations governing melting and melt solid interactions. A simple model for mass transfer during melting is proposed. In our model the temperature is eliminated as an independent variable by taking advantage of the fact that the Peclet number (characteristic spreading time/thermal diffusion time) is expected to be large beneath spreading centers. We present a linear instability analysis of localization in pure-shear which is tailored to represent deformation of a partly molten lithosphere in extension. Our analysis outlines two deformational regimes: melt-band dominated and shear-band dominated deformation. The nonlinear regime is explored in fully coupled finite element studies. The numerical analyses confirm the orientations of the localizations predicted in the linear instability analysis whereby the strength of the localization increases with increasing power law coefficient of the strain rate dependent viscosity. The influence of active melting turns out to be negligible for the order of strain rates considered here.
1 INTRODUCTION

The advent of plate tectonics some 50 years ago was the most important paradigm shift in geological research. Although plate kinematics is now broadly understood we still know relatively little about the way continents break up and oceans form. We have a principal understanding on the drivers and the movement of rigid plates, but we have, at best, a schematic understanding of the way mid-ocean ridges spread through segregation of melts [1]. Continental break up was flagged very early as a problem child of plate tectonics [2]. Yet, it is still an unsolved problem.

One key issue in extension, not yet fully understood, is the role played by melts, both for functioning of mid-ocean ridges and for the breakup of continents. Melt is arguably the lubricant that facilitates ocean separation, however, why is the mechanism so efficient at extremely low melt fraction inferred from REE chemistry [1] and why is it operating at extremely low strain rates inferred from plate kinematics? For continental deformation we can safely neglect the role of melts and investigate other localization mechanism driven by the solid rock matrix. However, at what stage in the process of ocean forming does melt become important during continental breakup? An understanding of the physics of these processes is crucial for predictive modeling of geodynamic processes.

Given the two end member behaviours in plate tectonics, deformable continents and rigid oceans, it appears that one could approach the plate tectonic deformation problem in two ways. One way would be to postulate that the focusing mechanism is entirely based on the role of melts. For an extension problem this would be the mid-ocean ridge end member. In this approach one could assume for sake of simplicity that melts are the active ingredient in the localization phenomenon and that localization phenomenon from the solid rock matrix play a minor role. For a continental extension problem, melts are not even required at the initial continental breakup. So the second approach would be the opposite and one could assume for simplicity that the melts are following some instability mechanism in the solid rock matrix. This approach is more generally valid since it does not presuppose the existence of melts. Both approaches have, however, their validity. To fully understand continental breakup and rifting processes, one would therefore need to identify: at what stage during continental extension do melts become an important ingredient, if at all. This is the question that we aim to resolve.

By way of elimination of basic localization mechanism we first address the problem from the end-member of melt segregation at mid-ocean ridges and neglect the solid deformational instabilities. In this paper we only investigate the problem of whether melts are a suitable candidate for efficient and sustained localization.

2 FORMULATION

We describe melt flow within a viscously deforming porous matrix based on the governing equations by McKenzie, 1984 [3]. The melt is assumed to form a homogeneous interconnected network distributed over length scales much larger than the grain radius but smaller than the geological length scales (kilometers).

2.1 Balance equations

Mass conservation for the solid and the melt requires:

\[(1-\phi)_{,t} + ((1-\phi)v_{,t}) = -\frac{\dot{m}}{\rho_i} \text{ and } \phi_{,t} + (\phi u_{,t}) = \frac{\dot{m}}{\rho_f} \tag{1}\]
where \( \mathbf{v} \) and \( \mathbf{u} \) designate the solid and the fluid velocity vector respectively; \( \phi \) is the melt porosity, the comma followed by a subscript designates partial differentiation with respect to the time \( t \) or a spatial coordinate; \( \rho_s \) and \( \rho_f \) are the solid and the fluid (melt) densities and \( \dot{m} = m_s + m_k v_k \) is the material rate of mass transfer between solid and melt, to be specified later. Adding solid to fluid balance gives:

\[
(\phi(u_k - v_k))_k = -v_{k,k} + \left( \frac{1}{\rho_f} - \frac{1}{\rho_s} \right) \dot{m}
\]  

Inserting Darcy’s law:

\[
\phi(u_i - v_i) = \frac{k(\phi)}{\eta^f} (p_s - \rho_f g_i)
\]  

into (2) yields:

\[
\left( \frac{k(\phi)}{\eta^f} (p_s - \rho_f g_k) \right)_k = v_{k,k} - \frac{\Delta \rho}{\rho_s \rho_f} \dot{m}
\]  

where \( k(\phi) \) is the permeability, \( \eta^f \) is the melt viscosity, the vector \( g_i \) is parallel to the direction of gravity and \( |g| = g \) where \( g \) is the gravitational constant and \( \Delta \rho = \rho_s - \rho_f \). With the excess pore pressure, (4) may be written as:

\[
\left( \frac{k(\phi)}{\eta^f} p_s \right)_k = v_{k,k} - \frac{\Delta \rho}{\rho_s \rho_f} \dot{m}
\]  

Neglecting inertia forces, the momentum balance is obtained as:

\[
(\eta(v_{i,j} + v_{j,i}) + (\eta_{s} - \frac{2}{3} \eta) v_{k,k} \delta_{ij})_j - P_{,j} + (1-\phi) \Delta \rho g_i = 0
\]  

Although the temperature does not occur explicitly in the model we are deriving here we wish to include the effect of viscous heating by adiabatic elimination of the temperature based on an assumed high Peclet number limit in the heat equation. The starting point for this derivation is an abridged form of the heat equation (e.g. [4]):

\[
\rho C_p \dot{T} = (kT)_x + L \dot{m} + \eta \dot{\gamma}^2 + \eta_p \dot{\varepsilon}^2
\]  

where \( \rho C_p = (1-\phi)(\rho C_p)_s + \phi(\rho C_p)_f \) is the density times the heat capacity at constant pressure, \( T \) is the temperature, the superscripted dot designates the material time derivative with respect to the solid material, \( L \) is the latent heat, \( k \) is the thermal conductivity and \( \dot{\varepsilon} = v_{k,k} \). Strictly speaking the material time derivative of the temperature should have been taken with respect to the fluid solid mixture, where the mixture velocity is defined as \( \mathbf{v}^m = \mathbf{v} + \phi(\mathbf{u} - \mathbf{v}) \). The second term of the mixture velocity was neglected here for convenience.

### 2.2 Constitutive relationships

The following expression for the shear viscosity was proposed by Kelemen et al. (1997; see also Zimmermann and Kohlstedt, 2004) [5,6]:

...
\[ \eta = \eta_{ref} e^{-\alpha(\phi - \phi_{ref})} \tag{8} \]

With \( \alpha = 25 \) in the Olivine+MORB system. The effect of strain rates is considered in a modification of (8) proposed by Katz et al (2006) as [7]:

\[ \eta = \eta_{ref} e^{-\alpha(\phi - \phi_{ref})} \left( \frac{\dot{\gamma}}{\dot{\gamma}_{ref}} \right)^{\frac{1}{n-1}} \tag{9} \]

Here, \( \dot{\gamma} = \sqrt{2D_{ij}D'_{ij}} \), where \( D_{ij} \) is the symmetric part of the solid velocity gradient and \( D'_{ij} = D_{ij} - 1/3D_{kk}\delta_{ij} \); \( \delta_{ij} \) is the Cartesian unit tensor. In 2D we have:

\[ \dot{\gamma} = 2\sqrt{1/3(D_{11}^2 - D_{11}D_{22} + D_{22}^2) + D_{12}^2} \tag{10} \]

For the bulk viscosity we assume (e.g. Mackenzie, 1950; see also Fig 6 in McKenzie, 1984) [8,3]:

\[ \eta_B = \frac{4\eta}{3\phi} \tag{11} \]

For the permeability we adopt the expression [3,4]:

\[ k(\phi) = k_{ref} \left( \frac{\phi}{\phi_{ref}} \right)^m, \quad k_{ref} = \frac{a^2}{b} \phi_{ref}^m \tag{12} \]

Following Schmeling’s (2000) discussion, \( a \) is defined as the scaling distance between melt inclusions (i.e. of the order of the grain size), \( b \) is a geometrical parameter between 100 and 3000) and \( m \) is a power exponent. In general power exponents always lie above 2 depending on the pore geometry. For example, for the idealized case of isotropically oriented fully connected melt tubules with equal circular cross sections the parameters are \( m = 2 \) and \( b = 72\pi \), while for isotropically oriented fully connected melt films the parameters are \( m = 3 \) and \( b = 162 \). If the interconnectivity of the melt is low at low melt fractions and increases with melt fraction, higher \( n \)-values are also expected [9]. We conclude this section with the derivation of a simple model for mass transfer by melting. The degree of melting at \((x,t)\) is defined as \( f = m/\rho_f \) with \( \dot{f} = m/\rho_f \) during melting where the dot designates the material time derivative of the solid [4]. For illustration we consider the simplified kinetic relationship:

\[ \dot{f} = 1/\tau^{melt}(F(p,T) - \bar{f}), \quad T_s \leq T \leq T_i \tag{13} \]

where \( F(p,T) \) is the equilibrium melt fraction corresponding to the pair \( p \) and \( T \) [1]. \( \tau^{melt} \) is the relaxation time of the melt process and \( T_s \) , \( T_i \) are the melt pressure dependent liquidus and solidus temperatures respectively. For processes operating on a time scale much larger than the melt relaxation time, i.e. if the timescale of interest \( \tau' \) is much larger than \( \tau^{melt} \), we may assume that \( f = \bar{F} \) and \( \dot{f} = \dot{\bar{F}} \). In this case the kinetic relationship becomes:

\[ \frac{\dot{m}}{\rho_f} = \frac{\partial F}{\partial T} \bar{T} + \frac{\partial F}{\partial p} \bar{p} \tag{14} \]

McKenzie and Bickle (1988) have derived an expression for the melt fraction \( X \) by weight for a given pair \( p \) and \( T \) [1]. The relationship reads:
\[ X = g \left( \frac{T - T_s}{T_t - T_s} \right), \quad X = \frac{f \rho_f}{(1 - f) \rho_s + f \rho_f}, \quad f = \frac{X \rho_s}{(1 - X) \rho_f + X \rho_s} \] (15)

The details of Eq. (15) are represented in appendix A for easy reference. The equation (15) is empirical and obtained by interpolation between \( X = 0 \) for \( g(0) \) and \( X = 1 \) for \( g(1) \). In our strain localization simulations we shall approximate \( F \) by \( X \).

We now introduce a number of simplifying assumptions to render our model more tractable, without distorting the essential physics of the processes we wish to model. In the preliminaries section of their paper McKenzie and Bickle (1988) note that the thermal Peclet number as \[\frac{v l}{k (\rho c_p)} (16)\]

\( Pe \) is obtained approximately equal to 30 (assuming \( v=10^{\text{mm/yr}} \) and \( l=100^{\text{km}} \)). The above equation shows that the temperature variation which controls melting beneath ridges is entirely governed by advection heat. In this case the heat equation (7) reduces to:

\[ T = \frac{1}{\rho c_p} \left( \eta \left( \dot{\gamma}^2 + \frac{4}{3} \dot{\varepsilon}^2 \right) - \rho_f L \dot{f} \right) \] (17)

If the timescale of interest is much larger than the melting relaxation time, (13) may be written as:

\[ \dot{\phi} = (1 - \phi) \dot{\varepsilon} + \frac{\rho_f}{\rho_s} \left( \frac{\partial F}{\partial T} T + \frac{\partial F}{\partial \dot{p}} \dot{p} \right) \] (18)

Since buoyancy effects are unimportant in the deformation and porosity localization problems considered here we shall also assume that \( \rho_s = \rho_f = \rho \) for convenience. Insertion of (18) and rearranging yields:

\[ \dot{\phi} = (1 - \phi) \dot{\varepsilon} + \frac{1}{1 + \frac{L \eta c_p}{\rho F}} \left( \frac{1}{\rho c_p} \eta \left( \dot{\gamma}^2 + \frac{4}{3} \dot{\varepsilon}^2 \right) + \frac{\partial F}{\partial \dot{p}} \dot{p} \right) \] (19)

We non-dimensionalise the governing equations (5), (6) and (19) with respect to time, space and pressure by defining:

\[ x_i = l_c x_i, \quad t = \tilde{t} \tilde{t}, \quad (P, p) = \eta_{ref} \left( \ddot{P}, \ddot{p} \right) \] (20)

where the compaction length \( l_c \) is defined as:

\[ l_c = \sqrt{\frac{\eta \eta ref + \frac{4}{3} \eta ref}{\eta_f}}, \quad \eta_{ref} = \frac{4 \eta ref}{\frac{3}{\phi_{ref}}} \] (21)

We also define the viscosity ratio, \( \chi \), as:
\[ \chi = \frac{\eta_{\text{ref}}}{\eta_{\text{ref}} + \frac{4}{3} \eta_{\text{ref}}} \]  

Inserting into (5), (6) and (11) and dropping tildes yields:

\[ \chi \left( \frac{\phi}{\phi_{\text{ref}}} \right)^m P_{,k,k} = v_{k,k} \]  

\[ (e^{-\alpha(\phi-\phi_{\text{ref}})}) \gamma^{1-n} \left( \gamma_{1,i,j} + v_{j,i,1} + \frac{2}{3} (\frac{\phi_{\text{ref}}}{\phi}) \gamma_{1,i,j} \right) - P_{,j} = 0 \]  

\[ \phi = (1-\phi)\dot{e} + a_T e^{-\alpha(\phi-\phi_{\text{ref}})} \gamma^{1-n} (\dot{\gamma}^2 + \frac{4}{3\phi} \dot{e}^2) + a_p (\dot{P} - \alpha_g v_g) \]  

Where the relative weight/contribution of the thermal, pressure and gravity effects/terms are controlled by \( a_T, a_p \) and \( \alpha_g \), respectively, as follows:

\[ a_T = \frac{\eta_{\text{ref}} \gamma_{\text{ref}}}{\rho c_p (1 + a_L) \partial F / \partial T}, \quad a_p = \frac{\eta_{\text{ref}} [GPa s] \gamma_{\text{ref}}}{1 + a_L} \frac{\partial F}{\partial p}, \quad \alpha_g = -\frac{\rho g l_c}{\eta_{\text{ref}} \gamma_{\text{ref}}} \]  

in which \( a_L = (L/c_p) \ast (\partial F / \partial T) \) and the dimensionless velocity component opposite to the direction of gravity is obtained as:

\[ v_g = -x_i g_i / g \gamma_{\text{ref}} l_c = -v_i g_i / g \gamma_{\text{ref}} l_c \]  

Typical values for the parameters relevant to our model are listed in table 1 below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density ( \rho )</td>
<td>3500</td>
<td>Kg/m(^3)</td>
</tr>
<tr>
<td>Latent heat</td>
<td>400</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>1300</td>
<td>J/(K kg)</td>
</tr>
<tr>
<td>Melt pressure (50 km depth)</td>
<td>1.5-2.0</td>
<td>GPa</td>
</tr>
<tr>
<td>Reference viscosity ( \eta_{\text{ref}} )</td>
<td>10(^{19})</td>
<td>Pas</td>
</tr>
<tr>
<td>Reference permeability ( k_{\text{ref}} )</td>
<td>10(^{-9}) for ( m=3 ) in (12)</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Fluid viscosity ( \eta_f )</td>
<td>10</td>
<td>Pas</td>
</tr>
<tr>
<td>Compaction length ( l_c )</td>
<td>100-1000</td>
<td>m</td>
</tr>
<tr>
<td>Reference strain rate ( \dot{\gamma}_{\text{ref}} )</td>
<td>10(^{-14})-10(^{-11})</td>
<td>1/s</td>
</tr>
<tr>
<td>( a_L )</td>
<td>0.1-1</td>
<td>----</td>
</tr>
<tr>
<td>Heat melt sensitivity ( a_T )</td>
<td>0.1(\times)10(^{-4})-0.4(\times)10(^{-1})</td>
<td>----</td>
</tr>
<tr>
<td>Pressure melt sensitivity ( a_p )</td>
<td>-0.2(\times)10(^{-4})-0.2(\times)10(^{-1})</td>
<td>----</td>
</tr>
<tr>
<td>(-\alpha_p \alpha_g )</td>
<td>1-100</td>
<td>----</td>
</tr>
<tr>
<td>( \partial F / \partial T )</td>
<td>0.5(\times)10(^{-3})-4(\times)10(^{-3})</td>
<td>K(^{-1})</td>
</tr>
<tr>
<td>( \partial F / \partial p )</td>
<td>-1.0</td>
<td>GPa(^{-1})</td>
</tr>
<tr>
<td>Viscosity ratio ( \chi )</td>
<td>0-0.2</td>
<td>----</td>
</tr>
</tbody>
</table>

Table 1: Table of parameters; the higher values correspond to \( T=T_s \) and the lower value to \( T=T_s+0.5(T_l-T_s) \) in Eq. (15).
3 LINEAR INSTABILITY ANALYSIS

We consider pure shear deformations of a rectangular block of material with sides parallel to \((x_1,x_2)\). We investigate the stability of this reference state by deriving a criterion determining conditions under which perturbations \(\delta \phi\), \(\delta v_i\), \(\delta P\) of the ground state variables \(\phi_{ref}\), \(v_{i,ref}\), \(P_{ref}\) increase (unstable) or decrease (stable). We assume that \(D_{11}^{ref} > 0\) where \(D_{22}^{ref} = -D_{11}^{ref}\) and \(P_{ref} = 0\). In the linear instability analysis the influence of gravity is ignored for simplicity; the influence of melting is also neglected here because of the smallness of the coefficients \(a_i\), \(a_T\) and \(a_P\) in Eq. 25 (cp. Table 1). The derivation is very similar to the one presented by Katz et al (2006) for the case of simple shear. We therefore restrict ourselves to a brief outline and refer to Katz et al for details [7]. First the governing equations (23-25) are linearised with respect to the increments. We then assume solutions of the hyperbolic type such as \(\delta \phi = F \exp(\omega t) \sin q(n_1 x_1)\), where \(q\) is the wave number, \(\omega\) is the growth coefficient and \((n_1,n_2) = (-\sin \beta, \cos \beta)\); the angle \(\beta\) is the angle between the positive \(x_1\) axis and the melt band. Insertion of the expressions for \(\delta \phi\), \(\delta v_i\), \(\delta P\) into the linearised governing equations yields the following expression (the so called dispersion relationship) for the dimensionless growth coefficient \(\tilde{\omega}\):

\[
\tilde{\omega} = \frac{-\chi \cos 2\beta}{1 + (1-\chi)(1-\frac{1}{n})\cos^2 2\beta + q^{-2}(1+\frac{1}{n}-1)\sin^2 2\beta}
\]

where \(\tilde{\omega} = \omega/|\alpha_{\phi,ref}(1-\phi_{ref})|\). We are looking for values of \((\beta,q)\) for which \(\tilde{\omega}\) assumes a maximum. The growth coefficient assumes a maximum with respect to \(q\) for \(q \to \infty\). For the special case \(n=1\) we obtain \(\tilde{\omega}(\beta, q \to \infty) = - \chi \cos 2\beta\) so that max \(\tilde{\omega}\) is obtained for \(\beta = \pi/2\), i.e. the band is oriented orthogonal to the principal extension axis.

![Figure 1: Deformation band growth rate as a function of orientation and rheology (strain-dependence exponent n) based on the dispersion relationship (Eq. 28) for n=1,2,3,4,6,10.](image)

This analytic result indicates that high growth rates (and hence strong localization) are only expected for materials with strong strain dependence (high \(n\) values). Materials with medium to low \(n\) levels \((1 < n < 4)\) show lower growth rates and less distinct peaks in the dispersion
curve (suggesting that there is no clear optimal orientation for localization). Finally, for \( n = 1 \), the distinct peak at \( \beta = 90 \) suggest that localization would occur at this orientation (albeit the low growth rate may result in a weak localization).

It should be noted that the maximum of the growth coefficient is obtained for \( \beta = \pi/2 \) for \( 1 \leq n \leq (2 - \chi)/(1 - \chi) \), while the \( \beta_{\text{max}} \) corresponding to max \( \tilde{\omega} \) is obtained from \( \cos 2\beta_{\text{max}} = (n(1 - \chi)(1 - 1/n))^{-1/2} \) for larger values of \( n \).

We conclude this section with an analytical solution for homogeneous, pure shear. The governing equations read:

\[
\tau = e^{-a(\phi - \phi_{\text{ref}})} \dot{\gamma}^{\frac{1}{n}}, \quad \dot{\phi} = a_i e^{-a(\phi - \phi_{\text{ref}})} \dot{\gamma}^{\frac{1}{n+1}}
\]

(29)

where we assume that \( \tau = (\sigma_{11} - \sigma_{22})/2 > 0 \) is given and constant. By eliminating \( \dot{\gamma} \) we obtain a first order differential equation for \( \phi \) with the solution:

\[
\phi - \phi_{\text{ref}} = -\ln(1 - a_r \alpha n \tau^{1+\alpha} t)
\]

(30)

The critical time for \( \phi \rightarrow \infty \) is obtained as:

\[
t_{\text{crit}} = \frac{1}{a_r \alpha n \tau^{1+\alpha}}
\]

(31)

The solution (30) may be useful for benchmarking finite element implementations of the present model.

A simple way to locate and quantify melting in mantle circulation simulations is to assume that the solid pressure is equal to the melt pressure. In this case the effective solid pressure \( p_r = -\eta_b v_{l,k} \) would be equal to zero. If the effective pressure disappears, the momentum balance equation (24) reduces to the standard Stokes equations for an incompressible fluid with a variable viscosity: \( p_r = -\eta_b v_{l,k} \). This case is obtained if either the compaction length \( l_c = \infty \) and/or \( \chi = 0 \) (eq.22). This assumption allows melt fractions and zones of melting to be identified in a post processing step. However this often effective strategy does not allow the characterization of melt instabilities or patterning since the growth coefficient \( \tilde{\omega} = 0 \) in (28) if \( \chi = 0 \).

4 FINITE ELEMENT SIMULATIONS

A rectangular block of initial dimensions \((L_0, D_0)\), constrained to plane deformations, is subjected to uniaxial extension with the principal extension axis parallel to the \( x_1 \) axis (Figure 2). Extension is superimposed onto an initial, purely hydrostatic state of effective stress (Figure 2). The initial excess melt pressure is assumed to be zero. The shear stresses are assumed to be zero on the block surfaces and a uniform horizontal velocity \( V_1 \) is prescribed on the sides \( x_1 = (0, L) \) in such a way that strain is constant at all times \( V_1/L = 1/2 \). The normal stresses due to extension on the top surface (initially at \( x_2 = H \) ) are assumed as zero, while \( v_2 = 0 \) at \( x_2 = 0 \). Since we use the excess melt pressure \( P \) instead of the melt pressure \( p \) in the numerical solution we have to apply the boundary condition \( P = \rho g l_c/(\eta_{\text{ref}} \dot{\gamma}_{\text{ref}}) \) on the free surface.
The changing geometry during deformation is considered by employing an updated Lagrangian integration scheme. The governing equations are solved numerically by means of the finite element based partial differential equation solver eScript [10]. Using an updated Lagrangian method, equation (25) is solved at the integration point level. The velocity and melt pressure equations (Eqn.23 and Eqn.24, respectively) are computationally decoupled by solving equation (24) first with \( P \) from the last iteration/time step whereby \( P \) is assumed equal to zero upon problem initiation. After Eqn.23 and Eqn.24 are solved we update the viscosities and the problem is resolved until convergence occurs. After convergence for the geometry at time \( t \), the mesh is updated and the problem is solved for \( t + \Delta t \).

Results are shown in Figure 3 for an initial mean porosity of \( \phi_{ref} = 0.1 \) with superimposed random fluctuations of \( \pm 0.01 \times \phi_{ref} \) shown for \( n = 1 \) (Fig. 3a) and \( n = 6 \) (Fig. 3b). The melt band orientations are orthogonal to the extension axis, as expected from the linear instability analysis for \( n = 1 \). For \( n = 6 \) we obtain \( \beta = 50^0 \) which is slightly less than \( 55^0 \) we would have expected from the linear instability analysis. The difference may be explained by the fact that in the linear instability analysis we have assumed a ground state in steady state with \( P = 0 \) and \( v_{k,k} = 0 \) whereas in the numerical simulation, the bands are evolving out of a flow with volumetric deformations and a non-uniformly distributed melt pressure.

The effective pressure distribution across the height of the block (Fig. 4a) and the sum of the effective and the melt pressure are shown in Figure 4b. The total pressure is constant across the height of the domain (Fig. 4c). As expected, with accumulated strain the model domain necks and the average stress in the middle of the domain becomes larger than that along the side boundaries (Fig. 4d).
the results show that the consideration of equilibrium melting (i.e. normalized elongation of the block for a range of values of the melt parameter $a_T$ in equation (25)) is insignificant for the reference strain rate and viscosity usually assumed for spreading centers. There is a potentially significant influence for higher rates and viscosities, on faults for instance in connection with the problem of deep earthquakes.

Figure 4: Dimensionless (a) pore (melt) pressure, (b) effective pore pressure, (c) total pressure (effective pressure plus melt pressure) vs. height in the centre on the domain, (d) average value of $\sigma_{11}$ over the cross sections in the middle ($X=L/2$) and on the boundary of the domain ($X=0$).

Figure 5 shows the average equivalent shear stress on $x_t = L$ for $n = 1$ as a function of the normalized elongation of the block for a range of values of the melt parameter $a_T$ in equation (25). The results show that the consideration of equilibrium melting (i.e. $a_T \neq 0$ in equation (25)) is insignificant for the reference strain rate and viscosity usually assumed for spreading centers. There is a potentially significant influence for higher rates and viscosities, on faults for instance in connection with the problem of deep earthquakes.

Figure 5: Average equivalent shear stress at $x_t=L$ for various values of the melting parameter $a_T$. In the simulations we have assumed that $a_p=0$ in Equation 25.
CONCLUSIONS

For ocean ridges the internal and external forces such as effective stress, melt pressure and slab-pull have been postulated to play an important role in melt localization and transport. A partially molten material undergoing deformation by shear is known to form a porosity localizing instability given that the viscosity of the matrix decreases with increase in melt fraction. Many theoretical, numerical and experimental analyses to explain and explore the properties of this instability have been carried out over the past years [1-8].

In the present contribution we gave an outline of the equations governing the mechanical behavior of a solid deforming viscously in a pressure and temperature regime in which partial melting occurs. We then conduct a linear instability analysis of a steady \((\phi = 0, v_{i,k} = 0)\) rectilinear deformation. The significance of the eigenmodes was subsequently explored in finite element studies based on the fully nonlinear model. In the analyses we assumed a nonlinear rheology incorporating melt fraction softening and power law strain rate hardening as proposed in [5-7]; we also derived specific expressions for the mass transfer rate \(\dot{m}\) in Eqn.(1).

In the linear instability analysis we concentrated on modes of the type \(\phi = \exp(i\omega t) \cdot f(q\sin \beta x_1 + \cos \beta x_2)\), where \(n_1^2 + n_2^2 = 1\), \(q\) is the wave number and \(\beta\) is the angle enclosed by the melt band axis and the positive \(x_1\) axis (see figure 3). The maximum of instability growth rate was obtained in the short wave length limit \(q \rightarrow \infty\) in all cases (independent of \(n, \beta\)). For \(1 \leq n \leq 2.13\) the maximum of \(\omega\) with respect to \(\beta\) occurred at \(\beta = \pi / 2\) while in the plasticity limit \(n \rightarrow \infty\), we recover \(\beta = \pm \pi / 4\). The growth coefficient \(\omega\) is quite small except for large \(n\) where the behavior is dominated by plasticity and not melting. The tendency to form porosity localizations under the conditions considered here is therefore weak. The orientations of porosity and strain localizations were predicted in fully-coupled, nonlinear, finite element simulations (Fig. 3). We note that the porosity in the localization bands is only about 30% higher than outside the bands for \(n = 1\). The strain concentration within the shear bands obtained for \(n > 3\) is much more significant and consistent with results familiar from simulations based on plasticity models [11].

The influence of active melting \((\dot{m} \neq 0)\) is negligible in the cases considered here assuming realistic values for the model parameters. This does not mean that active melting can always be neglected. Melting is an important weakening mechanism in the lower-crust and upper mantle [12-14], and it plays a central role in infiltration instabilities [15] and in plume related processes [16].

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REFERENCES


APPENDIX A

The melt volume fraction after McKenzie and Bickle (1988) assuming [1]:
where $a_1=2.0684$, $a_2=-4.0564$ and $a_3=2.988$. The solidus and liquidus functions are defined as:

$$T_s(p) = 1409. + 134.2 p - 6.581 p^2 + 0.1054 p^3$$  \hspace{1cm} (A2)$$

and

$$T_l(p) = 2035. + 57.46 p - 3.487 p^2 + 0.0769 p^3$$  \hspace{1cm} (A3)$$